



# ATMAM Mathematics Methods

Test 1 2019      Calculator Free

SHENTON  
COLLEGE

Name: .....

Teacher (Please circle name)      Ai      Friday      Smith

Time Allowed : 30 minutes

Marks	/32
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**Materials allowed:** Formula Sheet.

**Attempt all questions. Questions 1,2, 3,4 and 5 are contained in this section.**

**All necessary working and reasoning must be shown for full marks.**

*Where appropriate, answers should be given as exact values.*

*Marks may not be awarded for untidy or poorly arranged work.*

1.      [2,2,2,2]

Differentiate each of the following with respect to  $x$ , clearly showing the appropriate use of rules.  
Do not simplify answers.

(a)  $y = 4x^3 - \frac{1}{x^2} + \frac{1}{2}x$

(b)  $y = (3x + 2)^3(x^4 - 3)$

(c)  $y = \frac{\cos(3x+2)}{\sin x}$

(d)  $y = \sqrt{(5x - 4)}$

2. [4,6]

Consider the function  $f(x) = x^3(4 - x)$

- (a) Use calculus to determine the location of all stationary points.
- (b) Use the second derivative to determine the nature of the stationary points and the coordinates of any points of inflection.

3. [3 marks]

If  $y = 3 \sin 2x + 2 \cos 2x$  show that  $4y + \frac{d^2y}{dx^2} = 0$

4. [4 marks]

Determine  $\frac{dy}{dx}$  if  $y = \sqrt{u}$ ,  $u = v^2 + 1$  and  $v = x + x^{-1}$ . Do not simplify your answer.

5. [1,1,1,4]

The table below contains information about the sign of  $f(x)$ ,  $f'(x)$  and  $f''(x)$  at seven points on the graph of the continuous function  $f(x)$ . Apart from those in the table, there are no other points where  $f(x)$ ,  $f'(x)$  or  $f''(x)$  are equal to zero.

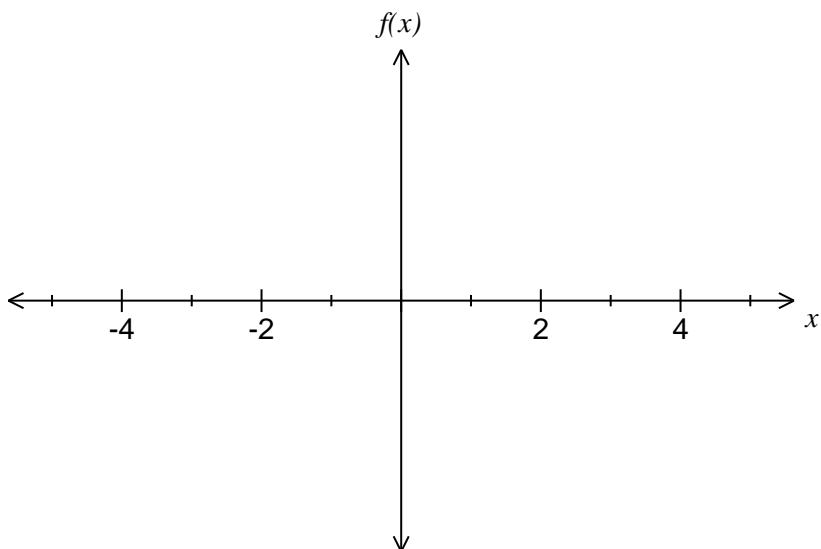
$x$	-3	-1	0	1	2	3	4
$f(x)$	-	0	+	+	+	0	-
$f'(x)$	+	0	+	+	0	-	-
$f''(x)$	-	0	+	0	-	-	-

(a) Describe the nature of the graph when  $x=2$

(b) At what value(s) of  $x$  is  $f(x)$  concave up?

(c) Describe the nature of the graph when  $x=-1$ .

(d) Sketch the function on the axes below.





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# ATMAM Mathematics Methods

## Test 1 2019      Calculator Assumed

Name: .....

Teacher (Please circle name)      Ai      Friday      Smith

Time Allowed : 20 minutes

Marks	/19
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**Materials allowed:** Classpad calculator, Formula Sheet.

**Attempt all questions. Questions 6, 7 and 8 are contained in this section.**

**All necessary working and reasoning must be shown for full marks.**

*Where appropriate, answers should be given as exact values.*

*Marks may not be awarded for untidy or poorly arranged work.*

6. [1,1,1,2]

A particle is moving in a straight line so that at time  $t$ , in seconds, its position from the origin  $O$  is given by  $x(t) = 7.2 - 3 \cos(0.65t)$  metres,  $t \geq 0$

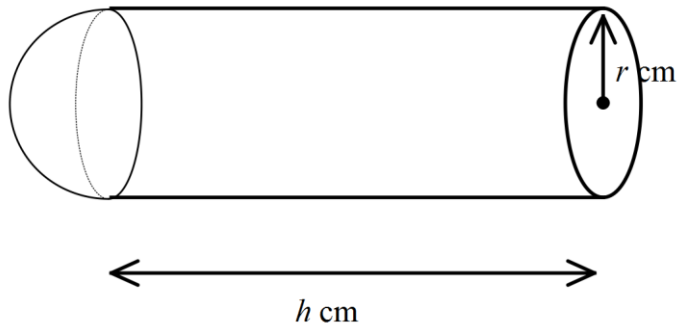
(a) State the initial position of the particle.

(b) Determine the velocity function for this particle.

(c) At what time does the particle first come to rest after  $t = 0$ ?

(d) At what time does the particle first reach its maximum velocity? Justify your choice.

7. [2,1,4,3]



A solid wooden peg consists of a cylinder of length  $h$  cm and a hemispherical cap of radius  $r$  cm. The volume,  $V$   $cm^3$ , of the peg is given by  $V = \pi r^2 h + \frac{2}{3} \pi r^3$ .

(a) If the surface area of the peg is  $100\pi$   $cm^2$ .

(i) Show that  $h = \frac{100 - 3r^2}{2r}$

(ii) Determine  $V$  as a function of  $r$ .

(iii) Show the use of calculus to determine the dimensions required to obtain the maximum volume and state the maximum volume.

(b) If  $h = 6 \text{ cm}$ , then  $V = 6\pi r^2 + \frac{2}{3}\pi r^3$ .

For  $r = 4 \text{ cm}$ ,

show that a small increase of  $k \text{ cm}$  in the radius results in an approximate increase of  $80\pi k \text{ cm}^3$  in the volume.

8. [4 marks]

If  $y = 5t^3$  use differentiation to determine the approximate percentage change in  $y$  when  $t$  changes by 4%.